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Instantons and Monopoles in Maximal Abelian Projection of $SU(2)$ Gluodynamics

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Abstract

We show that the instantons induce the abelian monopoles in the Maximal Abelian Projection of $SU(2)$ gluodynamics. As an example we consider the case of one instanton and the case of a set of instantons arranged along a straight line. The abelian monopoles which are induced by instantons may play some role in the confinement scenario.

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1 Introduction

The confinement phenomena in QCD is one of the most important problems in the nonabelian theories. Various instanton models of QCD yield vanishing string tension for the QCD string [1], however, the instantons seem to play an important role. For instance the correlation functions of various colourless quark bound states are well described by the instanton contributions [2].

The explanation of the confinement phenomena was proposed by 't Hooft [3] and Mandelstam [4], who conjecture that the infrared properties of confining QCD vacuum are similar to those of the (dual) superconductor. The most convenient way to think about this analogy is to partially fix the $SU(N)$ gauge degrees of freedom leaving $[U(1)]^{N-1}$ group unfixed [5]. Such partial gauge is usually called abelian projection. Under the abelian transformations, the diagonal elements of the gluon field transform as gauge fields and due to the compactness of the $U(1)$ gauge group, the abelian monopoles exist. If they are condensed, the string between the coloured charges is formed as the dual analogue of the Abrikosov string in a superconductor, the monopoles playing the role of the Cooper pairs [3, 4].

Many numerical experiments demonstrate the dual superconductor mechanism in $SU(2)$ lattice gauge theory (for a review see [6]); the most part of the results was obtained in so called Maximal Abelian (MA) projection [7]. However, there exists the Abelian projection in which the role of monopoles is played by the other topological objects - by the "minopoles" [8].

Below we show that several instantons in the 't Hooft anzatz, with the centers placed on the straight line, lead to the abelian monopole current along this line. For the sake of simplicity we discuss only the $SU(2)$ gauge group.

2 From Instanton to Abelian Monopole

The $SU(2)$ instanton field configuration in the 't Hooft anzatz is given by the equation

$$A_\mu^a = \frac{1}{g} \bar{\eta}_{\mu\nu}^a \partial_\nu f(x), \quad (1)$$

where $\bar{\eta}_{\mu\nu}^a$ is 't Hooft symbol and g is coupling constant. The function $f(x)$ is given by

$$f(x) = f^I(x) = \ln \left[1 + \frac{\rho^2}{t^2 + r^2} \right], \quad (2)$$

where ρ is the size of the instanton, t – time, r – spatial radius.

The gauge conditions which define the MA projection are given by the formula [7]:

$$(\partial_\mu \pm ig A_\mu^3) A_\mu^\pm = 0, \quad (3)$$

where $A_\mu^\pm = A_\mu^1 \pm i A_\mu^2$, the $SU(2)$ generators are $t^a = \sigma^a/2$ and σ^a are the Pauli matrices. Let us rotate the instanton field (1,2) by the $SU(2)$ matrix Ω ,

$$\Omega = \begin{pmatrix} \cos \phi e^{i\theta} & \sin \phi e^{i\chi} \\ -\sin \phi e^{-i\chi} & \cos \phi e^{-i\theta} \end{pmatrix}, \quad (4)$$

where

$$\chi = \Delta - \alpha/2, \quad \theta = -\Delta - \alpha/2, \quad \phi = \gamma/2. \quad (5)$$

Here α and γ are azimuthal and polar angles of the reference system in the given time-slice, $\Delta(x)$ is an arbitrary function. As it can be easily checked, the instanton field A which is rotated by this matrix Ω

$$A_\mu \rightarrow A_\mu^{(\Omega)} = \Omega^+ A_\mu \Omega - \frac{i}{g} \Omega^+ \partial_\mu \Omega, \quad (6)$$

satisfies the MA projection conditions (3).

The field strength tensor

$$G_{\mu\nu}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu] \quad (7)$$

with the transformed field $A_\mu^{(\Omega)}$ satisfies the equation

$$\epsilon_{\mu\nu\alpha\beta} \partial_\nu G_{\alpha\beta}[A^{(\Omega)}] = \frac{2\pi}{g} j_\mu \cdot \sigma^3, \quad (8)$$

where j_μ is given by the following equation:

$$j_\mu(x) = \delta^{(3)}(\mathbf{r}) \cdot \delta_{\mu 0}. \quad (9)$$

The equation (8) is nothing but the definition of the monopole singularities in the gauge field A_μ^3 , since the only third component $G^3[A^{(\Omega)}]$ contributes¹ to the *r.h.s* of eq.(8). The monopole current $j_\mu(x)$ in eq.(9) is the straight line along the time-direction which crosses the center of the instanton (which is the center of our reference system).

Note, that the monopole trajectory does not depend on the choice of the function $\Delta(x)$, which was used in the definition of the matrix Ω , eqs.(4) and (5). As it can be shown, this function corresponds to the residual $U(1)$ degrees of freedom we leave unfixed. $\Delta(x)$ does not affect $U(1)$ -invariant quantities while $U(1)$ -variant quantities (*e.g.*, the position of the Dirac string which is attached to the abelian monopole) changes if the function $\Delta(x)$ is changed.

The choice (5) for the $SU(2)$ -angles θ , χ and ϕ is not unique. Let us choose an arbitrary unit vector n_μ and define three vectors $m_\mu^{(a)}$, $a = 1, 2, 3$ as follows:

$$m_\mu^{(k)} = \bar{\eta}_{\mu\nu}^a n_\nu. \quad (10)$$

Using the properties of the 't Hooft symbols $\bar{\eta}_{\mu\nu}^a$ we can easily derive that vectors n and $m^{(a)}$ form the basis in $4D$ Euclidean space-time:

$$m_\mu^{(k)} m_\mu^{(l)} = \delta^{kl}; \quad m_\mu^{(k)} n_\mu = 0 \quad k = 1, 2, 3. \quad (11)$$

¹Note, that the field strength tensor (7) transforms under singular gauge transformations Ω as follows: $G_{\mu\nu} \rightarrow G_{\mu\nu}[A^{(\Omega)}] = \Omega^+ G_{\mu\nu}[A] \Omega + G_{\mu\nu}^{sing}[\Omega]$, where $G_{\mu\nu}^{sing}[\Omega] = -i\Omega^+(x)[\partial_\mu \partial_\nu - \partial_\nu \partial_\mu]\Omega(x)$. The matrix (4) is singular and therefore $G_{\mu\nu}^{sing}[\Omega]$ is not zero. It can be also shown that only $G_{\mu\nu}^{sing}[\Omega]$ contributes to the *r.h.s* of eq.(8).

Let us choose the $SU(2)$ -angles θ , χ and ϕ as in eq.(5) but with the azimuthal, α , and polar, γ , angles are defined with respect to the new 3D basis $\{m_\mu^{(a)}\}$. We call the matrix Ω , eq.(4), with new angles θ , χ and ϕ as $\Omega[n]$. It can be easily verified that the transformation of the instanton field (1,2) by the matrix $\Omega[n]$ satisfies the MA projection conditions (3).

Repeating all the above calculations, but with new matrix Ω we get for the abelian monopole current which has to be defined from eq.(8):

$$j_\mu = \delta^{(3)}(\mathbf{y}) \cdot n_\mu, \quad (12)$$

where $y^k = m_\nu^k x_\nu$. The monopole trajectory is the straight line which crosses the center of the instanton and has the direction n_μ .

3 Two and Many Instantons

The N -instanton field configuration $A^{N,I}$ may be written in the 't Hooft ansatz (1), where the function $f(x)$ is given by the formula:

$$f(x) = f^{N,I}(x) = \ln \left[1 + \sum_{i=1}^N \frac{\rho_i^2}{(x - x_i)^2} \right]. \quad (13)$$

Here x_i is the position of the center of i -th instanton, all the instantons have the same colour orientations.

Consider first the case of two instantons, $N = 2$. Let us rotate the field (1,13) by the matrix $\Omega[n]$, where the vector n_μ is oriented along the line which connects the centers x_1 and x_2 of these instantons. The direct evaluation shows that the rotated field satisfies the MA projection conditions (3). Substituting the field $A = A^{N,I}$ and the matrix $\Omega = \Omega[n]$ into eq.(8) we finally get for the abelian monopole current the expression (12). Note, that the sizes ρ_i of the considered instantons are not important.

Thus we can draw the conclusion: any two instantons in the 't Hooft ansatz bear in the MA projection the straight-line abelian monopole current j_μ which goes through the centers of these instantons. Note that we did not prove the uniqueness of the solution of the equation (3) with respect to the gauge transformations (6), the existence of the other monopoles trajectories are not excluded.

The already described matrix $\Omega[n]$ rotates an arbitrary configuration of instantons within the MA projection, provided that the centers of all these instantons are on the same straight line and the colour orientations are also the same. As in the two-instanton case, the sizes of instantons are irrelevant and the resulting abelian monopole trajectory goes through the centers of instantons.

The particular case of the considered instanton configuration corresponds to the BPS-monopole [9]: the distance between any two neighbour centres should be fixed and the sizes of the all instantons should be equal to infinity. The BPS-monopole trajectory (which is the straight line) coincides with the trajectory of the abelian monopole. This case was already discussed in ref. [10].

There are more general field configurations which give in the MA projection, the abelian monopole trajectories which are arbitrary straight lines with the direction n_μ . These field configurations may be represented in the 't Hooft ansatz (1) and the function $F(x)$ being dependent only on the combinations $t' = n \cdot x$ and $r' = \sqrt{x^2 - (n \cdot x)^2}$. The resulting monopole lines cross the center of our reference system.

4 Conclusions

Our calculations show that the single instanton field configuration can be rotated within the MA projection by various gauge transformations each of which is parametrized by arbitrary vector n_μ . The induced monopole currents are the straight lines with the direction n_μ . Many-instanton configurations induce abelian monopoles as well.

The calculations in the lattice gluodynamics show, that the abelian monopoles are responsible for the confinement in the MA projection [6]. The contribution of the abelian monopoles to the Wilson loop leads to the area law with the correct string tension coefficient [11, 12]. Moreover, it was observed [13] that the long monopole loops alone are responsible for the behavior of the string tension in the confinement phase of $SU(2)$ QCD. We found that the monopole trajectories of this type may be induced by instantons (and BPS monopoles) into MA projected $SU(2)$ vacuum. Therefore the monopoles induced by instantons may in principle be important for the confinement scenario.

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References

- [1] C.G. Callan Jr., R.F. Dashen, D.J. Gross, *Phys. Rev.*, **D 17** (1978) 2717;
D.I. Diakonov and V.Yu. Petrov, *Phys. Lett.*, **226B** (1989) 372.
- [2] M.C. Chu, J.M. Grandy, S. Huang, J.W. Negele, *Phys. Rev.*, **D49** (1994) 6039;
Proceedings of the symposium Lattice 93, *Nucl. Phys.*, **B** (Proc.Suppl.) **34** (1994) 170.
- [3] G. 't Hooft, in 'High Energy Physics', ed. M. Zichichi (Editrice Compositori, Bologna, 1976).
- [4] S. Mandelstam, *Phys. Rep.*, **23C** (1976) 245.
- [5] G. 't Hooft, *Nucl. Phys.*, **B190[FS3]** (1981) 455.
- [6] T. Suzuki, *Proceedings of the symposium Lattice 92*, *Nucl. Phys.*, **B** (Proc.Suppl.) **30** (1993) 176.
- [7] A.S. Kronfeld, G. Schierholz and U.-J. Wiese, *Nucl. Phys.*, **B293** (1987) 461;
A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.-J. Wiese, *Phys. Lett.*, **198B** (1987) 516.
- [8] M.N. Chernodub, M.I. Polikarpov and A.I. Veselov, *Phys. Lett.*, **B342** (1995) 303.
- [9] E.B. Bogomol'nyi, *Sov. J. Nucl. Phys.*, **24** (1976) 449.
M. K. Passard and C. M. Sommerfeld, *Phys. Rev. Lett.*, **35** (1975) 760.
- [10] Jan Smit, Arjan van der Sijs, *Nucl. Phys.*, **B355**(1991) 603.
- [11] J.D. Stack, S.D. Neiman and R.J. Wensley, *Phys. Rev.*, **D50** (1994) 3399.
- [12] H. Shiba and T. Suzuki, *Phys. Lett.*, **B333** (1994) 461.
- [13] S. Ejiri, S. Kitahara, Y. Matsubara and T. Suzuki, *Phys. Lett.*, **B343**(1995) 304.